Apologia for A Failed Counter-Example Refutation of Lorentz Transformations of Special Relativity

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1 Author's Apologia

I regret having published an error plagued earlier version of this paper as a refutation of special relativity. Once the errors were scrubbed out of it, there was no refutation in it. I am sorry for the inconvenience and distraction it may have occasioned you.

You may wonder why I look for a refutation of special relativity. It is reason enough that science requires ongoing vigilance for error in our beliefs. In the case of special relativity in particular, the theory is unusual in having been crafted on the basis of imagined rather than actual experiment. There was no empirical basis for the theory.

While some believe it was inspired or based on the Michelson Morley experiment, it was not. Einstein subsequently declared that he had heard of but was not familiar with the Michelson Morley experiment at the time. Special relativity denies the meaningfulness of absolute rest and absolute velocity; it deals with comparison of clocks and relatively moving frames of reference, while the Michelson Morley experiment involves no clock and only one frame of reference, (unless an absolute rest frame of reference is admitted.)

Further, Einstein's special relativity paper, though appealing because it is more readable than most scientific papers, is fraught with logical fallacy, mathematical error, contradictory statements, assertions without basis, ambiguity and equivocation. It is remarkable for so compromised a work to be considered a theory at all. Claims of verification are often attributable to equivocation, misinterpretation or favorable bias in interpreting ambiguities of the theory. For example, the slowing of a clock is presented as an appearance due to the relative movement of the clock and the observer, not as a change intrinsic to the clock due to its velocity. Remember, absolute velocity is denied.

So biased has the physics community become in favor of Einstein's theory that they accept some empirical refutations of special relativity as confirmations of the theory. For example, Willem de Sitter's astronomical proof of the constancy of light speed, sometimes cited as a confirmation, showed that the speed of light from a binary star to earth is constant independent of the varying velocity of the star, i. e. not in the frame of reference of the star; whereas special relativity claims light speed to be constant relative to every reference frame.

Confirmation bias is the enemy of science. To enlighten your understanding of special relativity, try reading "On the Electrodynamics of Moving Bodies" while holding in mind the thought that its author may have been a technobabbling nit wit.

Michelson, de Sitter, Ives (cited as one to confirm time dilation), and others did not accept special relativity. Ludwig Silberstein, an early promoter of relativity, ultimately rejected it. There are many respectable relativity skeptics today, it is reprehensible to claim that it is beyond question.

I shall continue my quest.

2 Abstract

An essential claim of special relativity is that the Lorentz transformation equations convert spacetime coordinates of an event relative to one inertial frame of reference to the corresponding spacetime coordinates of the same event relative to a second inertial frame of reference in uniform motion relative to the first. If this is so, it must be possible to again transform from the second to a third frame of reference and thence back to the first frame of reference producing there a match to the original coordinates. A mathematical calculation that does not rely on any empirical data tests this claim. The consistency of the Lorentz transformations is verified in this instance.

3 Summary

The Lorentz transformation requires that the x-axis of the first reference frame's coordinate system be parallel to the relative velocity of the second. This will require a rotation of coordinates between successive transformations.

We consider three inertial frames of reference, designated **ONE**, **TWO** and **THREE**. Origins coincide at time $t = 0$, as required for the transformation.

To reveal the relative velocities, we specify with respect to **ONE** the subsequent space-time coordinates of each origin, with $\mathbf{O_{1,2}}$, (coordinates with respect to **ONE** of the origin of **TWO**,) on the x-axis and $\mathbf{O_{1,3}}$ in the xy-plane of frame **ONE**. We also specify space-time coordinates of an event, $\mathbf{E_1}$. We transform each to frame **TWO**. Then we rotate coordinates about the z-axis of **TWO** to align its x-axis with the relative velocity of **THREE**. We transform to **THREE** and rotate to align **THREE**'s x-axis with $\mathbf{O_1}$. Upon rotating coordinates to restore the original orientation of **ONE**, the originally given space-time coordinates are regenerated.

4 Transformations and Rotations

4.1 Conventions for Units and Subscripts

We make the second our unit of time and the light-second our unit of distance. This simplifies computations by making $c = 1$

As we change from one frame of reference to another, the changes of coordinate values can be disorienting. We shall use subscripts to keep our bearings; the first subscript will designate the frame of reference, the second subscript denotes an object, (e. g. $v_{m,n}$ is velocity of n with respect to m, and $x_{n,k}$ is the x-coordinate of k in the coordinate system of frame n.) Coordinates that have been rotated will have an r appended in the subscript, and regenerated coordinates will have an a or ar appended to the subscript for the frame of reference.

4.2 Lorentz Transformation Equations

The Lorentz transformation equations for velocity in the direction of the x-axis with corresponding axes parallel, simplified using $c = 1$, are:

$$x_{n,k} = \gamma_{m,n}(x_{m,k} - v_{m,n}t_{m,k}) \tag{1}$$

$$y_{n,k} = y_{m,k} \tag{2}$$

$$z_{n,k} = z_{m,k} \tag{3}$$

$$t_{n,k} = \gamma_{m,n}(t_{m,k} - v_{m,n}x_{m,k}) \tag{4}$$

where

$$\gamma_{m,n} = \frac{1}{\sqrt{1 - v_{m,n}^2}} \tag{5}$$

4.3 Rotation Matrices

The matrix for rotating space-time coordinates about the z-axis, leaving time unchanged has the form

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{6}$$

with

$$\sin\theta = \frac{y}{\sqrt{x^2+y^2}} \tag{7}$$

$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}} \tag{8}$$

where x and y are initial coordinates of the point to be aligned with the rotated x-axis.

Matrix calculator utilities are available on the internet, e. g. matrixcalc.org.

5 Initial Conditions

We consider three inertial frames of reference, designated **ONE**, **TWO** and **THREE**, with origins coinciding at time $t = 0$.

At time $t = 1$, let the vector specification $\begin{pmatrix} x & y & z & t \end{pmatrix}$ of the origins with respect to **ONE** be as follows:

The origin of **ONE** is,

$$\mathbf{O_{1,1}} = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix} \tag{9}$$

The origin of **TWO** is,

$$\mathbf{O_{1,2}} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 1 \end{pmatrix} \tag{10}$$

The origin of **THREE** is,

$$\mathbf{O_{1,3}} = \begin{pmatrix} 0 & \frac{1}{3} & 0 & 1 \end{pmatrix} \tag{11}$$

Transforming these origin coordinates from frame of reference **ONE** to **TWO**, thence to **THREE**, and back to **ONE**, will be accomplished without discrepancy. However, the corresponding transformation of an event **E** not coinciding with an origin fails to regenerate the original coordinates with respect to **ONE**. Our example uses the event with frame **ONE** coordinates,

$$\mathbf{E_1} = \begin{pmatrix} 3 & 5 & 4 & 2 \end{pmatrix} \tag{12}$$

6 Transforming Observations to Frame TWO

Now, let us use Lorentz transformations to find coordinates with respect to **TWO**.

6.1 Parameters of the Transformation

From the coordinates of the origin of **TWO**,

$$\mathbf{O_{1,2}} = \left(\begin{array}{cccc} \frac{1}{2} & 0 & 0 & 1 \end{array} \right) \tag{10}$$

we find the relative velocity

$$v_{1,2} = \frac{x_{1,2}}{t_{1,2}} = \frac{1}{2} \tag{13}$$

Evaluating $\gamma_{1,2}$ for $v_{1,2} = \frac{1}{2}$,

$$\gamma_{1,2} = \frac{1}{\sqrt{1 - \left(\frac{1}{2}\right)^2}} = \frac{2\sqrt{3}}{3} \tag{14}$$

6.2 Origin of ONE to Frame TWO

Transforming the origin of **ONE** from **ONE**, at time $t = 1$, to **TWO**,

$$\mathbf{O_{1,1}} = \left(\begin{array}{cccc} 0 & 0 & 0 & 1 \end{array} \right) \tag{9}$$

$$x_{2,1} = \frac{2\sqrt{3}}{3}\left(0 - \frac{1}{2} \times 1\right) = \frac{-\sqrt{3}}{3} \tag{15}$$

$$y_{2,1} = 0 \tag{16}$$

$$z_{2,1} = 0 \tag{17}$$

$$t_{2,1} = \frac{2\sqrt{3}}{3}\left(1 - \frac{1}{2} \times 0\right) = \frac{2\sqrt{3}}{3} \tag{18}$$

we find the origin of **ONE** in **TWO**

$$\mathbf{O_{2,1}} = \left(\begin{array}{cccc} \frac{-\sqrt{3}}{3} & 0 & 0 & \frac{2\sqrt{3}}{3} \end{array} \right) \tag{19}$$

6.3 Origin of TWO to Frame TWO

Transforming the origin of **TWO** from **ONE**, at time $t = 1$, to **TWO**,

$$\mathbf{O_{1,2}} = \left(\begin{array}{cccc} \frac{1}{2} & 0 & 0 & 1 \end{array} \right) \tag{10}$$

$$x_{2,2} = \frac{2\sqrt{3}}{3}\left(\frac{1}{2} - \frac{1}{2} \times 1\right) = 0 \tag{20}$$

$$y_{2,2} = 0 \tag{21}$$

$$z_{2,2} = 0 \tag{22}$$

$$t_{2,2} = \frac{2\sqrt{3}}{3}\left(1 - \frac{1}{2} \times \frac{1}{2}\right) = \frac{\sqrt{3}}{2} \tag{23}$$

we find the origin of **TWO** in **TWO**

$$\mathbf{O_{2,2}} = \left(\begin{array}{cccc} 0 & 0 & 0 & \frac{\sqrt{3}}{2} \end{array} \right) \tag{24}$$

6.4 Origin of THREE to Frame TWO

Transforming the origin of **THREE** from **ONE**, at time $t = 1$, to **TWO**,

$$\mathbf{O_{1,3}} = \left(\begin{array}{cccc} 0 & \frac{1}{3} & 0 & 1 \end{array} \right) \tag{11}$$

$$x_{2,3} = \frac{2\sqrt{3}}{3}\left(0 - \frac{1}{2} \times 1\right) = \frac{-\sqrt{3}}{3} \tag{25}$$

$$y_{2,3} = \frac{1}{3} \tag{26}$$

$$z_{2,3} = 0 \tag{27}$$

$$t_{2,3} = \frac{2\sqrt{3}}{3}\left(1 - \frac{1}{2} \times 0\right) = \frac{2\sqrt{3}}{3} \tag{28}$$

we find the origin of **THREE** in **TWO**

$$\mathbf{O_{2,3}} = \left(\begin{array}{cccc} \frac{-\sqrt{3}}{3} & \frac{1}{3} & 0 & \frac{2\sqrt{3}}{3} \end{array} \right) \tag{29}$$

6.5 The Event to Frame TWO

Transforming the event from **ONE** to **TWO**,

$$\mathbf{E_1} = \begin{pmatrix} 3 & 5 & 4 & 2 \end{pmatrix} \tag{12}$$

$$x_{2,E} = \frac{2\sqrt{3}}{3}\left(3 - \frac{1}{2} \times 2\right) = \frac{4\sqrt{3}}{3} \tag{30}$$

$$y_{2,E} = 5 \tag{31}$$

$$z_{2,E} = 4 \tag{32}$$

$$t_{2,E} = \frac{2\sqrt{3}}{3}\left(2 - \frac{1}{2} \times 3\right) = \frac{\sqrt{3}}{3} \tag{33}$$

we find the event in frame **TWO**

$$\mathbf{E_2} = \begin{pmatrix} \frac{4\sqrt{3}}{3} & 5 & 4 & \frac{\sqrt{3}}{3} \end{pmatrix} \tag{34}$$

7 Transforming Observations to Frame THREE

7.1 Parameters of the Transformation

We find velocity from the coordinates of the origin of **THREE**,

$$\mathbf{O_{2,3}} = \begin{pmatrix} \frac{-\sqrt{3}}{3} & \frac{1}{3} & 0 & \frac{2\sqrt{3}}{3} \end{pmatrix} \tag{29}$$

$$\sqrt{\left(\frac{-\sqrt{3}}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + 0^2} = \frac{2}{3}$$

$$v_{2,3} = \frac{\frac{2}{3}}{\frac{2\sqrt{3}}{3}} = \frac{\sqrt{3}}{3} \tag{35}$$

Evaluating $\gamma_{2,3}$ for $v_{2,3} = \frac{\sqrt{3}}{3}$,

$$\gamma_{2,3} = \frac{1}{\sqrt{1 - \left(\frac{\sqrt{3}}{3}\right)^2}} = \frac{\sqrt{6}}{2} \tag{36}$$

7.2 Aligning x-axis of TWO with the Relative Velocity of THREE

The sine and cosine of the angle θ of the relative velocity $v_{2,3}$ with the original x-axis are found from coordinates of the origin of **THREE**

$$\mathbf{O_{2,3}} = \begin{pmatrix} \frac{-\sqrt{3}}{3} & \frac{1}{3} & 0 & \frac{2\sqrt{3}}{3} \end{pmatrix} \tag{29}$$

$$\sin\theta = \frac{y}{\sqrt{x^2+y^2}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2} \tag{37}$$

$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}} = \frac{\frac{-\sqrt{3}}{3}}{\frac{2}{3}} = \frac{-\sqrt{3}}{2} \tag{38}$$

These are used to form a rotation matrix $\mathbf{M_{2,3}}$ to align the x-axis of **TWO** with the relative velocity of **THREE**

$$\mathbf{M_{2,3}} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{-\sqrt{3}}{2} & \frac{-1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{-\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{39}$$

We effect rotation of frame **TWO** coordinates by applying $\mathbf{M_{2,3}}$ to each coordinate vector.

7.3 Origin of ONE to Frame THREE

7.3.1 Rotating in TWO

Rotating coordinates of the origin of **ONE** from (19)

$$\mathbf{O_{2r,1}} = \mathbf{O_{2,1}}\mathbf{M_{2,3}} = \begin{pmatrix} \frac{-\sqrt{3}}{3} & 0 & 0 & \frac{2\sqrt{3}}{3} \end{pmatrix} \begin{pmatrix} \frac{-\sqrt{3}}{2} & \frac{-1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{-\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{6} & 0 & \frac{2\sqrt{3}}{3} \end{pmatrix} \tag{40}$$

7.3.2 Transforming the Origin of ONE from TWO to THREE

By transforming rotated coordinates of the origin of **ONE** to **THREE**...

$$x_{3,1} = \frac{\sqrt{6}}{2}\left(\frac{1}{2} - \frac{\sqrt{3}}{3} \times \frac{2\sqrt{3}}{3}\right) = \frac{-\sqrt{6}}{12} \tag{41}$$

$$y_{3,1} = \frac{\sqrt{3}}{6} \tag{42}$$

$$z_{3,1} = 0 \tag{43}$$

$$t_{3,1} = \frac{\sqrt{6}}{2}\left(\frac{2\sqrt{3}}{3} - \frac{\sqrt{3}}{3} \times \frac{1}{2}\right) = \frac{3\sqrt{2}}{4} \tag{44}$$

we find the origin of **ONE** in **THREE**

$$\mathbf{O_{3,1}} = \begin{pmatrix} \frac{-\sqrt{6}}{12} & \frac{\sqrt{3}}{6} & 0 & \frac{3\sqrt{2}}{4} \end{pmatrix} \tag{45}$$

7.4 Origin of TWO to Frame THREE

7.4.1 Rotating in TWO

Rotating coordinates of the origin of **TWO** in **TWO** is trivial, from (24).

$$\mathbf{O_{2r,2}} = \mathbf{O_{2,2}} = \begin{pmatrix} 0 & 0 & 0 & \frac{\sqrt{3}}{2} \end{pmatrix} \tag{24}$$

7.4.2 The Origin of TWO from TWO to THREE

Transforming rotated coordinates of the origin of **TWO** to frame **THREE**

$$x_{3,2} = \frac{\sqrt{6}}{2}\left(0 - \frac{\sqrt{3}}{3} \times \frac{\sqrt{3}}{2}\right) = \frac{-\sqrt{6}}{4} \tag{46}$$

$$y_{3,2} = 0 \tag{47}$$

$$z_{3,2} = 0 \tag{48}$$

$$t_{3,2} = \frac{\sqrt{6}}{2}\left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times 0\right) = \frac{3\sqrt{2}}{4} \tag{49}$$

we find the origin of **ONE** in **THREE**

$$\mathbf{O_{3,2}} = \begin{pmatrix} \frac{-\sqrt{6}}{4} & 0 & 0 & \frac{3\sqrt{2}}{4} \end{pmatrix} \tag{50}$$

7.5 Origin of THREE to Frame THREE

7.5.1 Rotating in TWO

Rotating coordinates of the origin of **THREE** from (29)

$$\mathbf{O_{2r,3}} = \mathbf{O_{2,3}}\mathbf{M_{2,3}} = \begin{pmatrix} \frac{-\sqrt{3}}{3} & \frac{1}{3} & 0 & \frac{2\sqrt{3}}{3} \end{pmatrix} \begin{pmatrix} \frac{-\sqrt{3}}{2} & \frac{-1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{-\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & 0 & 0 & \frac{2\sqrt{3}}{3} \end{pmatrix} \tag{51}$$

7.5.2 The Origin of THREE from TWO to THREE

Transforming rotated coordinates of the origin of **THREE** to **THREE**,

$$x_{3,3} = \frac{\sqrt{6}}{2}\left(\frac{2}{3} - \frac{\sqrt{3}}{3} \times \frac{2\sqrt{3}}{3}\right) = 0 \tag{52}$$

$$y_{3,3} = 0 \tag{53}$$

$$z_{3,3} = 0 \tag{54}$$

$$t_{3,3} = \frac{\sqrt{6}}{2}\left(\frac{2\sqrt{3}}{3} - \frac{\sqrt{3}}{3} \times \frac{2}{3}\right) = \frac{2\sqrt{2}}{3} \tag{55}$$

we find the origin of **THREE** in **THREE**.

$$\mathbf{O_{3,3}} = \begin{pmatrix} 0 & 0 & 0 & \frac{2\sqrt{2}}{3} \end{pmatrix} \tag{56}$$

7.6 The Event to Frame THREE

7.6.1 Rotating in TWO

Rotating coordinates of the Event from (34)

$$\mathbf{E_{2r}} = \mathbf{E_2}\mathbf{M_{2,3}} = \begin{pmatrix} \frac{4\sqrt{3}}{3} & 5 & 4 & \frac{\sqrt{3}}{3} \end{pmatrix} \begin{pmatrix} \frac{-\sqrt{3}}{2} & \frac{-1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{-\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{-19\sqrt{3}}{6} & 4 & \frac{\sqrt{3}}{3} \end{pmatrix} \tag{57}$$

7.6.2 The Event from TWO to THREE

Transforming rotated coordinates of the Event to **THREE**,

$$x_{3,E} = \frac{\sqrt{6}}{2}\left(\frac{1}{2} - \frac{\sqrt{3}}{3} \times \frac{\sqrt{3}}{3}\right) = \frac{\sqrt{6}}{12} \tag{58}$$

$$y_{3,E} = \frac{-19\sqrt{3}}{6} \tag{59}$$

$$z_{3,E} = 4 \tag{60}$$

$$t_{3,E} = \frac{\sqrt{6}}{2}\left(\frac{\sqrt{3}}{3} - \frac{\sqrt{3}}{3} \times \frac{1}{2}\right) = \frac{\sqrt{2}}{4} \tag{61}$$

we find the coordinates of the Event in **THREE**.

$$\mathbf{E_3} = \begin{pmatrix} \frac{\sqrt{6}}{12} & \frac{-19\sqrt{3}}{6} & 4 & \frac{\sqrt{2}}{4} \end{pmatrix} \tag{62}$$

8 Transforming Observations to Frame ONE

8.1 Parameters of the Transformation

We find velocity from the coordinates of the origin of **ONE** in **THREE**,

$$\mathbf{O_{3,1}} = \begin{pmatrix} \frac{-\sqrt{6}}{12} & \frac{\sqrt{3}}{6} & 0 & \frac{3\sqrt{2}}{4} \end{pmatrix} \tag{45}$$

$$\sqrt{\left(\frac{-\sqrt{6}}{12}\right)^2 + \left(\frac{\sqrt{3}}{6}\right)^2} = \frac{\sqrt{2}}{4}$$

$$v_{3,1} = \frac{\frac{\sqrt{2}}{4}}{\frac{3\sqrt{2}}{4}} = \frac{1}{3} \tag{63}$$

Which is also apparent from the Coordinates of **THREE** in **ONE**, $\left(0, \frac{1}{3}, 0, 1\right)$.

Evaluating $\gamma_{3,1}$ for $v_{3,1} = \frac{1}{3}$,

$$\gamma_{3,1} = \frac{1}{\sqrt{1 - \left(\frac{1}{3}\right)^2}} = \frac{3\sqrt{2}}{4} \tag{64}$$

8.2 Aligning x-axis of THREE with the Relative Velocity of ONE

The sine and cosine of the angle θ of the relative velocity $v_{3,1}$ with **THREE**'s original x-axis are

$$\sin\theta = \frac{y}{\sqrt{x^2+y^2}} = \frac{\frac{\sqrt{3}}{6}}{\frac{\sqrt{2}}{4}} = \frac{\sqrt{6}}{3} \tag{65}$$

$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}} = \frac{\frac{-\sqrt{6}}{12}}{\frac{\sqrt{2}}{4}} = \frac{-\sqrt{3}}{3} \tag{66}$$

These are used to form a rotation matrix to align the x-axis with the relative velocity.

$$\mathbf{M_{3,1}} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{-\sqrt{3}}{3} & \frac{-\sqrt{6}}{3} & 0 & 0 \\ \frac{\sqrt{6}}{3} & \frac{-\sqrt{3}}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{67}$$

We effect this rotation of frame **TWO** coordinates by applying $\mathbf{M_{3,1}}$ to each coordinate vector.

8.3 Origin of ONE to Frame ONE

8.3.1 Rotating in THREE

Rotating coordinates of the origin of **ONE** from (45)

$$\mathbf{O_{3r,1}} = \mathbf{O_{3,1}}\mathbf{M_{3,1}} = \begin{pmatrix} \frac{-\sqrt{6}}{12} & \frac{\sqrt{3}}{6} & 0 & \frac{3\sqrt{2}}{4} \end{pmatrix} \begin{pmatrix} \frac{-\sqrt{3}}{3} & \frac{-\sqrt{6}}{3} & 0 & 0 \\ \frac{\sqrt{6}}{3} & \frac{-\sqrt{3}}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{4} & 0 & 0 & \frac{3\sqrt{2}}{4} \end{pmatrix} \tag{68}$$

8.3.2 Transforming the Origin of ONE from Rotated THREE to ONE

Transforming rotated coordinates of **ONE** to frame **ONE**,

$$x_{1a,1} = \frac{3\sqrt{2}}{4}\left(\frac{\sqrt{2}}{4} - \frac{1}{3} \times \frac{3\sqrt{2}}{4}\right) = 0 \tag{69}$$

$$y_{1a,1} = 0 \tag{70}$$

$$z_{1a,1} = 0 \tag{71}$$

$$t_{1a,1} = \frac{3\sqrt{2}}{4}\left(\frac{3\sqrt{2}}{4} - \frac{1}{3} \times \frac{\sqrt{2}}{4}\right) = 1 \tag{72}$$

we find the coordinates of the origin of **ONE** in **ONE**

$$\mathbf{O_{1a,1}} = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix} \tag{73}$$

8.4 Origin of TWO to Frame ONE

8.4.1 Rotating in THREE

Rotating coordinates of the origin of **TWO** from (50)

$$\mathbf{O_{3r,2}} = \mathbf{O_{3,2}M_{3,1}} = \begin{pmatrix} \frac{-\sqrt{6}}{4} & 0 & 0 & \frac{3\sqrt{2}}{4} \end{pmatrix} \begin{pmatrix} \frac{-\sqrt{3}}{3} & \frac{-\sqrt{6}}{3} & 0 & 0 \\ \frac{\sqrt{6}}{3} & \frac{-\sqrt{3}}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{4} & \frac{1}{2} & 0 & \frac{3\sqrt{2}}{4} \end{pmatrix} \tag{74}$$

8.4.2 Transforming the Origin of TWO from Rotated THREE to ONE

Transforming rotated coordinates of **TWO** to frame **ONE**,

$$x_{1a,2} = \frac{3\sqrt{2}}{4}\left(\frac{\sqrt{2}}{4} - \frac{1}{3} \times \frac{3\sqrt{2}}{4}\right) = 0 \tag{75}$$

$$y_{1a,2} = \frac{1}{2} \tag{76}$$

$$z_{1a,2} = 0 \tag{77}$$

$$t_{1a,2} = \frac{3\sqrt{2}}{4}\left(\frac{3\sqrt{2}}{4} - \frac{1}{3} \times \frac{\sqrt{2}}{4}\right) = 1 \tag{78}$$

we find the coordinates of the origin of **TWO** in **ONE**

$$\mathbf{O_{1a,2}} = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 1 \end{pmatrix} \tag{79}$$

8.5 Origin of THREE to Frame ONE

8.5.1 Rotating in THREE

The rotation of the origin of **THREE** in **THREE** is trivial

$$\mathbf{O_{3r,3}} = \mathbf{O_{3,3}} = \begin{pmatrix} 0 & 0 & 0 & \frac{2\sqrt{2}}{3} \end{pmatrix} \tag{80}$$

8.5.2 Transforming the Origin of THREE to ONE

Transforming the origin of **THREE** to frame **ONE**,

$$x_{1a,3} = \frac{3\sqrt{2}}{4}\left(0 - \frac{1}{3} \times \frac{2\sqrt{2}}{3}\right) = \frac{-1}{3} \tag{81}$$

$$y_{1a,3} = 0 \tag{82}$$

$$z_{1a,3} = 0 \tag{83}$$

$$t_{1a,3} = \frac{3\sqrt{2}}{4}\left(\frac{2\sqrt{2}}{3} - \frac{1}{3} \times 0\right) = 1 \tag{84}$$

we find the coordinates of the origin of **THREE** in **ONE**

$$\mathbf{O_{1a,3}} = \begin{pmatrix} \frac{-1}{3} & 0 & 0 & 1 \end{pmatrix} \tag{85}$$

8.6 The Event to Frame ONE

8.6.1 Rotating Coordinates of the Event in THREE

$$\mathbf{E_{3r}} = \mathbf{E_3 M_{3,1}} = \begin{pmatrix} \frac{\sqrt{6}}{12} & \frac{-19\sqrt{3}}{6} & 4 & \frac{\sqrt{2}}{4} \end{pmatrix} \begin{pmatrix} \frac{-\sqrt{3}}{3} & \frac{-\sqrt{6}}{3} & 0 & 0 \\ \frac{\sqrt{6}}{3} & \frac{-\sqrt{3}}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{-13\sqrt{2}}{4} & 3 & 4 & \frac{\sqrt{2}}{4} \end{pmatrix} \tag{86}$$

8.6.2 Transforming the Event from Rotated THREE to ONE

Transforming the event to frame **ONE**,

$$x_{1a,E} = \frac{3\sqrt{2}}{4}\left(\frac{-13\sqrt{2}}{4} - \frac{1}{3} \times \frac{\sqrt{2}}{4}\right) = -5 \tag{87}$$

$$y_{1a,E} = 3 \tag{88}$$

$$z_{1a,E} = 4 \tag{89}$$

$$t_{1a,E} = \frac{3\sqrt{2}}{4}\left(\frac{\sqrt{2}}{4} - \frac{1}{3} \times \frac{-13\sqrt{2}}{4}\right) = 2 \tag{90}$$

we find the coordinates of the origin of **TWO** in **ONE**

$$\mathbf{E_{1a}} = \begin{pmatrix} -5 & 3 & 4 & 2 \end{pmatrix} \tag{91}$$

9 The Rotation of the Regenerated Frame ONE to the Original Orientation

Originally, the origin of **TWO** was on the x-axis of **ONE**. From the regenerated coordinates of the origin of **TWO** with respect to **ONE**

$$\begin{pmatrix} 0 & \frac{1}{2} & 0 & 1 \end{pmatrix} \tag{79}$$

we find the rotation matrix.

$$\mathbf{M_{1,2}} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{92}$$

9.1 Regeneration of Original Coordinates

$$\mathbf{O_{1ar,1}} = \mathbf{O_{1a,1}}\mathbf{M_{1,2}} = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix} \tag{93}$$

The origin of **ONE** is successfully regenerated.

$$\mathbf{O_{1ar,2}} = \mathbf{O_{1a,2}}\mathbf{M_{1,2}} = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 1 \end{pmatrix} \tag{94}$$

The origin of **TWO** is successfully regenerated.

$$\mathbf{O_{1ar,3}} = \mathbf{O_{1a,3}}\mathbf{M_{1,2}} = \begin{pmatrix} \frac{-1}{3} & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{3} & 0 & 1 \end{pmatrix} \quad (95)$$

The origin of **THREE** is successfully regenerated.

$$\mathbf{O_{1ar,E}} = \mathbf{O_{1a,E}}\mathbf{M_{1,2}} = \begin{pmatrix} -5 & 3 & 4 & 2 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 5 & 4 & 2 \end{pmatrix} \quad (96)$$

Coordinates of the event are successfully regenerated.

It appears that the Lorentz transformations yield consistent results when applied to three or more frames of refererence with non-parallel relative velocities.

Elsewhere, under the title "Introduction to FitzGerald Relativity," an alternative theory is proposed.